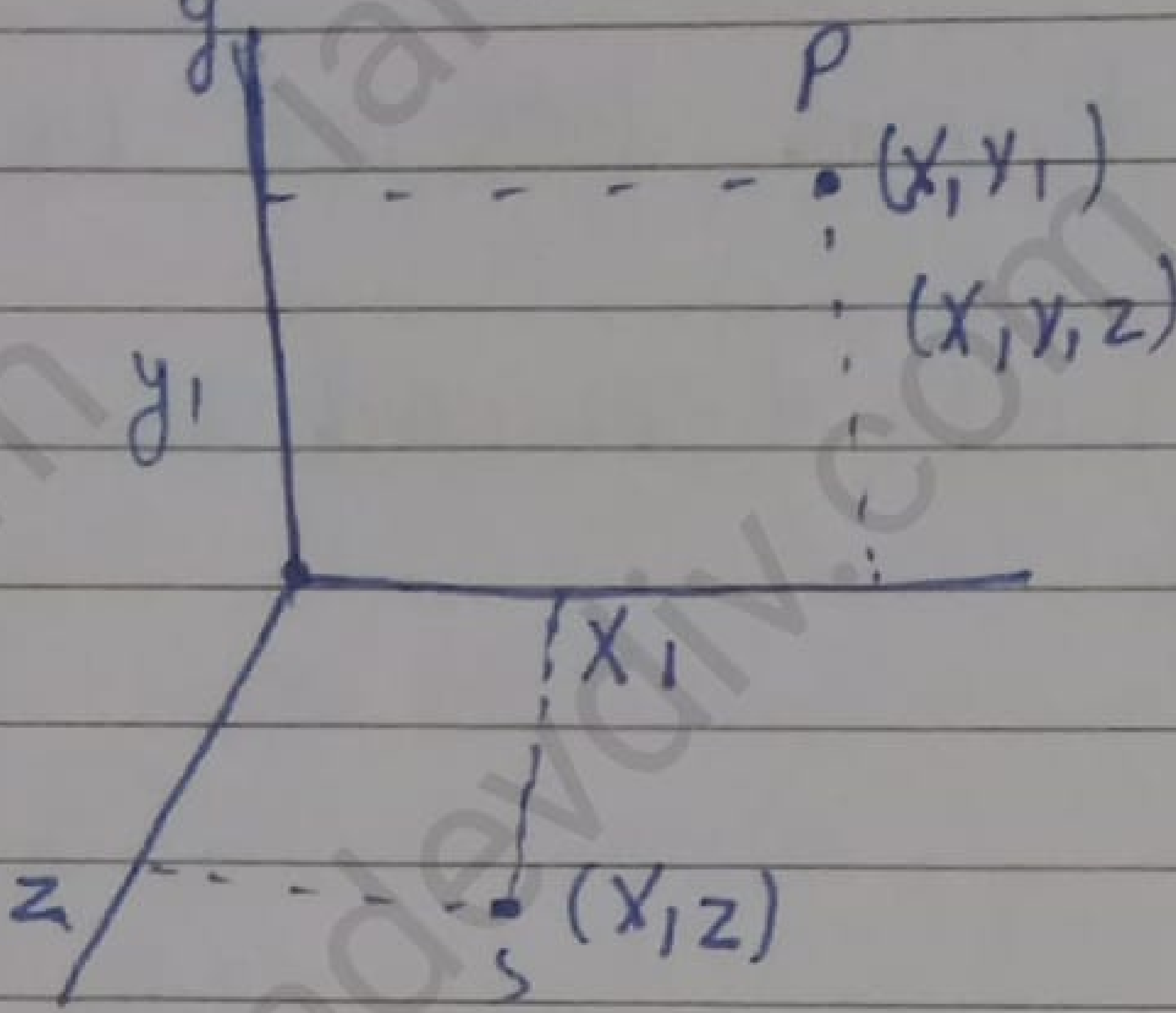
5/May/2023

Chapter - 3

Motion in Straight line

Frame of Refrence (Refrence frame)



1 D - Plane OX, OY, OZ → Motion 1 DM

2 D - Plane XY, YZ, ZX → Motion → 2 DM

3 D - Plane XYZ → Motion → 3 DM movement

(X, y), (X, y, z) (x_2, z_2) Coordinate in the position

↓

Position

→ **Speed**:- Speed of an object in motion is the ratio of total path length to the corresponding time taken by the object

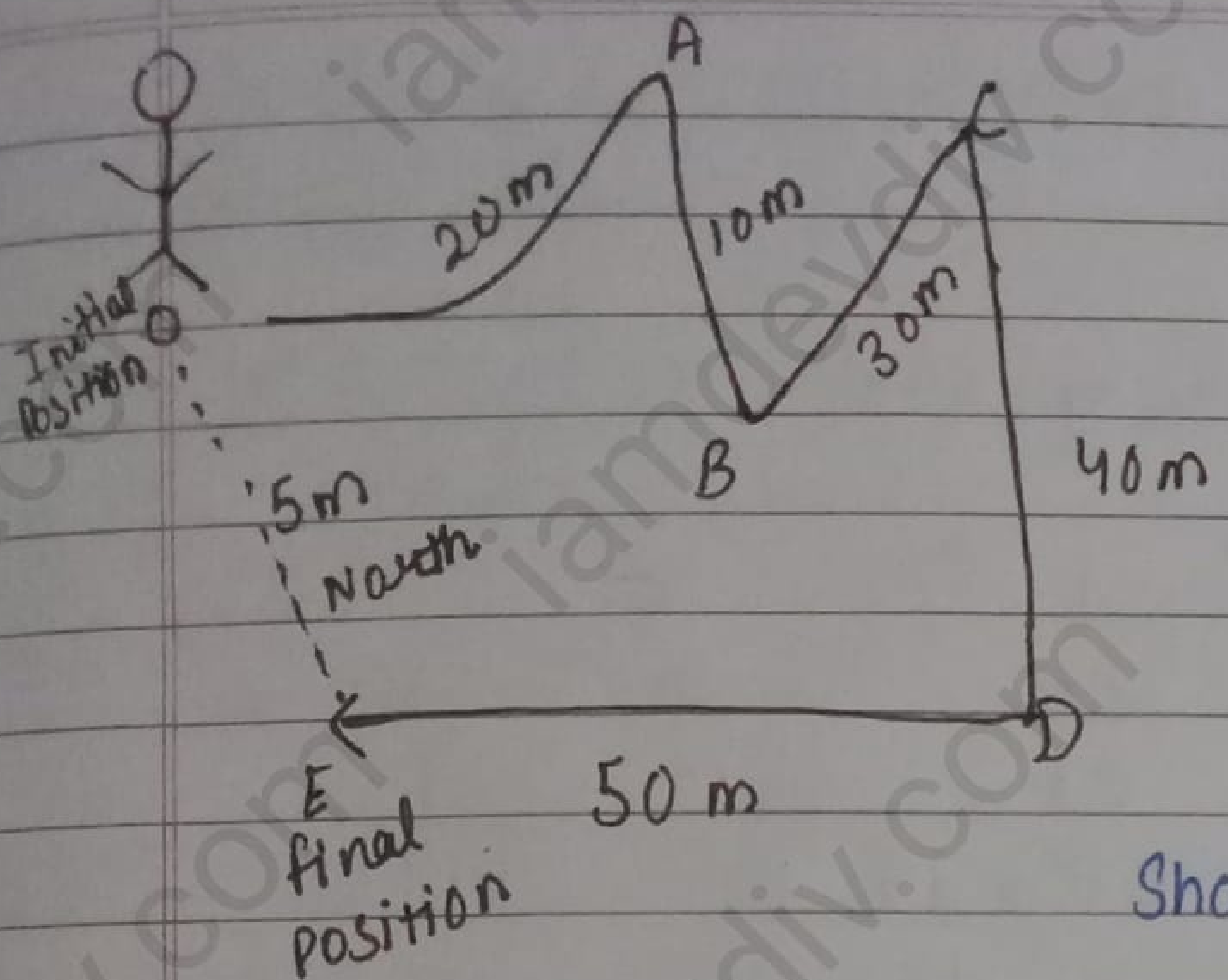
$$\text{Speed} = \frac{\text{total Path length}}{\text{time taken}}$$

- **Uniform speed**:- An object is said to be moving with uniform speed, if it covers equal distances in equal intervals of time

- **Average speed**:- It is the ratio of the total distance travelled by the body of the total time taken

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

- **Instantaneous speed**:- It is the speed of an object at a given instant of time.



IP ——— FP (Total covered path)

O ——→ E

OA + AB + BC + CD + DE
= 20 + 10 + 30 + 40 + 50
= 150 m

Shortest covered path

OE = 5m in North

The shortest covered path from I.P. to f.P. its a vector quantity. (displacement)

scalare → distance

$\vec{S}$ (direction)	S
↓	↓
Displacement	Distance

* Magnitude
* Direction
* May be zero

* Magnitude
* No direction
* Never zero

Distance = displacement, during the motion of any object along straight line

Ex:-

O ——— B

$\vec{OB} = OB$

Page No
Date / /20

#

$\vec{v}$ speed | $\vec{v}$ velocity

$v = \frac{d}{dt} s$ — $\vec{v} = \frac{d}{dt}\vec{s}$

OR — OR

$v = \frac{d}{dt} x$ — $\vec{v} = \frac{d\vec{x}}{dt}$

$$\boxed{\begin{array}{c} m/s \\ [M^0LT^{-1}] \end{array}}$$

$\vec{x} = 30t^4 + 4t^3 + 4t^2$

$t = 2$ sec

$\vec{v} = ?$

$$\vec{v} = \frac{d}{dt}(30t^4 + 4t^3 + 4t^2)$$

$$= 30\frac{d}{dt}t^4 + 4\frac{d}{dt}t^3 + 4\frac{d}{dt}t^2$$

$$\vec{v} = 120t^3 + 12t^2 + 8t \quad \text{—(1)}$$

$$= 120(2)^3 + 12(2)^2 + 8(2)$$

$$= 960 + 48 + 16$$

$$= 1024 \text{ m/s}$$

To find out acceleration we again take differentiation of eq (1)

$\vec{x} = 30t^4 + 4t^3 + 4t^2$

$t = 2$ sec.

$a = \frac{d}{dt}\vec{v} = \frac{d}{dt}(120t^3 + 12t^2 + 8t)$

$= 120\frac{d}{dt}t^3 + 12\frac{d}{dt}t^2 + \frac{8d}{dt}d$

$= 360(2)^2 + 24(2) + 8 = 1440 + 48 + 8$

$\Rightarrow \cancel{36}\ 1496\ m/s^2$ (Ans)

\# $\vec{x} = 40t^4 + 16t^3 + 2t^2$

$\vec{v} = ?$ $t = 3$ sec.

$\vec{a} = ?$

$\frac{d}{dt}(40t^4 + 16t^3 + 2t^2)$

$= 40\frac{d}{dt}t^4 + 16\frac{d}{dt}t^3 + 2\frac{d}{dt}t^2\}$

$\vec{v} = 160t^3 + 48t^2 + 4t$

$= 160(3)^3 + 48(3)^2 + 4(3)$

$= 1320\ \cancel{1320+}\ 432 + 12$

$= 1764$

$$\vec{a} = \frac{d}{dt}\left(160t^3 + 48t^2 + 4t\right) \qquad \int t^n\,dt = \frac{t^{n+1}}{n+1}$$

$$\vec{a} = 160\cdot 3t^2 + 48\cdot 2t + 4$$
$$= 480t^2 + 96t + 4$$
$$= 480\times 9 + 96\times 3 + 4$$
$$= 4320 + 198 + 4$$
$$= 4612\ m/s^2$$

\# $\vec{V} = 2t^3 + 6t^2 + 4t$

$t = 2\ sec$

$\vec{x} = ?$

$$\frac{d\vec{x}}{dt} = 2t^3 + 6t^2 + 4t$$

$$\int d\vec{x} = 2\int t^3\,dt + 6\int t^2\,dt + 4\int t\,dt$$

$$\vec{x} = \cancel{2}\frac{t^4}{\cancel{4}_{2}} + \cancel{6}^{2}\frac{t^3}{\cancel{3}} + \cancel{4}^{2}\frac{t^2}{\cancel{2}}$$

$$= \frac{t^4}{2} + 2t^3 + 2t^2$$

$$= \frac{2^4}{2} + 2(2)^3 + 2(2)^2 \Rightarrow \frac{\cancel{16}^{8}}{\cancel{2}} + 2(8) + 8 = 32\ m/s$$

\# $\vec{a} = 3t^2 + 4t$

$t = 2$ Sec.

$\vec{v} = ?$

$\vec{x} = ?$

$\frac{d\vec{x}}{dt} = 3t^2 + 4t$ $\quad \vec{v} = \int \vec{a}\, dt$

$\int d\vec{x} = 3\int t^2\, dt + 4\int t$

$= \frac{\cancel{3}^{1} t^3}{\cancel{3}} + \frac{\cancel{4}^{2} t^2}{\cancel{2}}$

$= (2)^3 + 2(2)^2$

$= 8 + 8$

$= 16$

$\frac{d\vec{x}}{dt} = t^3 + 2t^2$

$\int \frac{d\vec{x}}{dt} = \int t^3 + 2t^2$

$\vec{x} = \frac{t^4}{4} + \frac{2t^3}{3}$

$\vec{x} = 4 + \frac{16}{3}$

$\vec{x} = \frac{12+16}{3} = \frac{28}{3} = 9.33\,m$

Equation of motion for a uniformly accelerated motion

1. $v = u + at$

2. $S = at + \frac{1}{2}at^2$

3. $v^2 = u^2 + 2aS$

4. $S_n = u_n + \frac{1}{2}a(2n-1)$

- u = Initial velocity
- v = Final velocity
- a = acceleration
- t = Time
- S = distance
- S_n = distance per second
- $(2n-1)$ = Time

→ **Velocity:-** velocity of an object is the ratio of displacement and the corresponding time interval taken by the object.

$$\text{velocity} = \frac{\text{displacement}}{\text{Time taken}}$$

It is a vector quantity. SI unit of velocity is ms^{-1}. Its dimensional formula is $[M^0 L^1 T^{-1}]$

- **Uniform velocity:-** An object is said to be moving with uniform velocity, if it undergoes equal displacements in equal intervals of time, however small these intervals may be.
- **Instantaneous velocity:-** The velocity of an object at any given instant of time is known as instantaneous velocity.

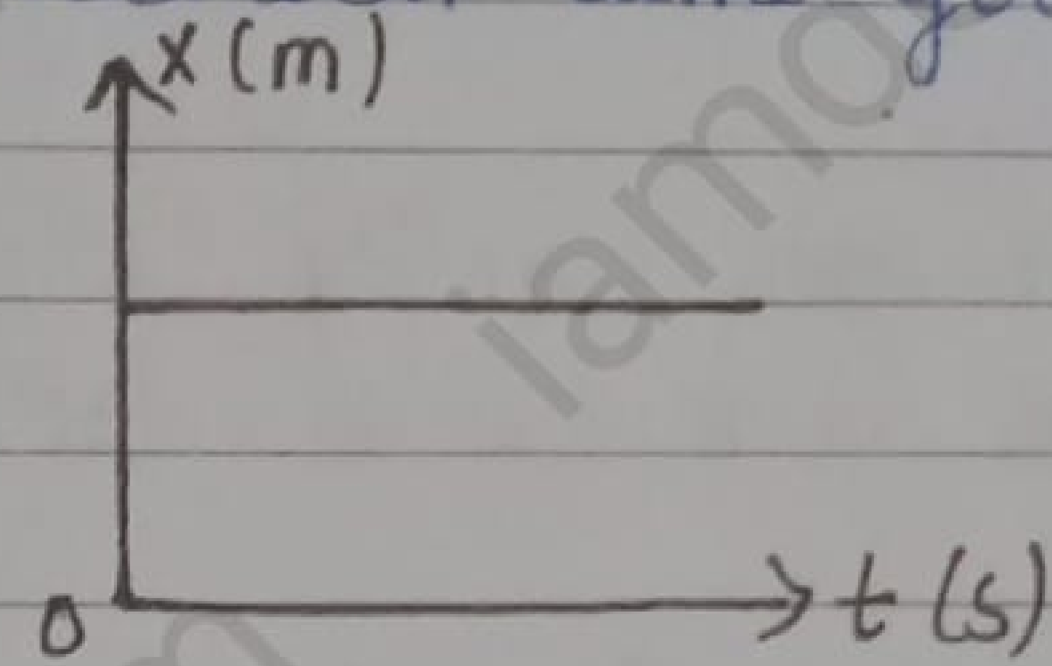
$$= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- **Average velocity**:- It is the ratio of displacement to the time interval for which the motion takes place.

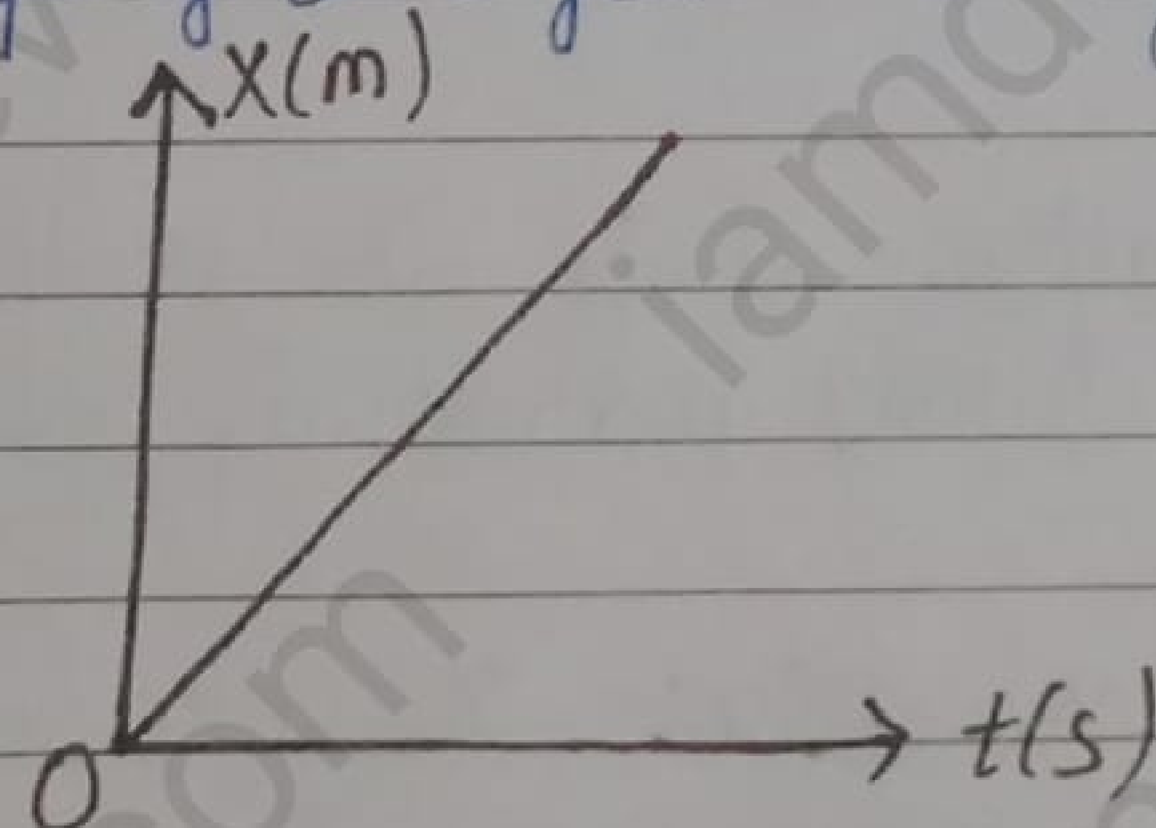
$$\text{Average velocity} = \frac{\text{displacement}}{\text{Time taken}}$$

* Position - Time Graph

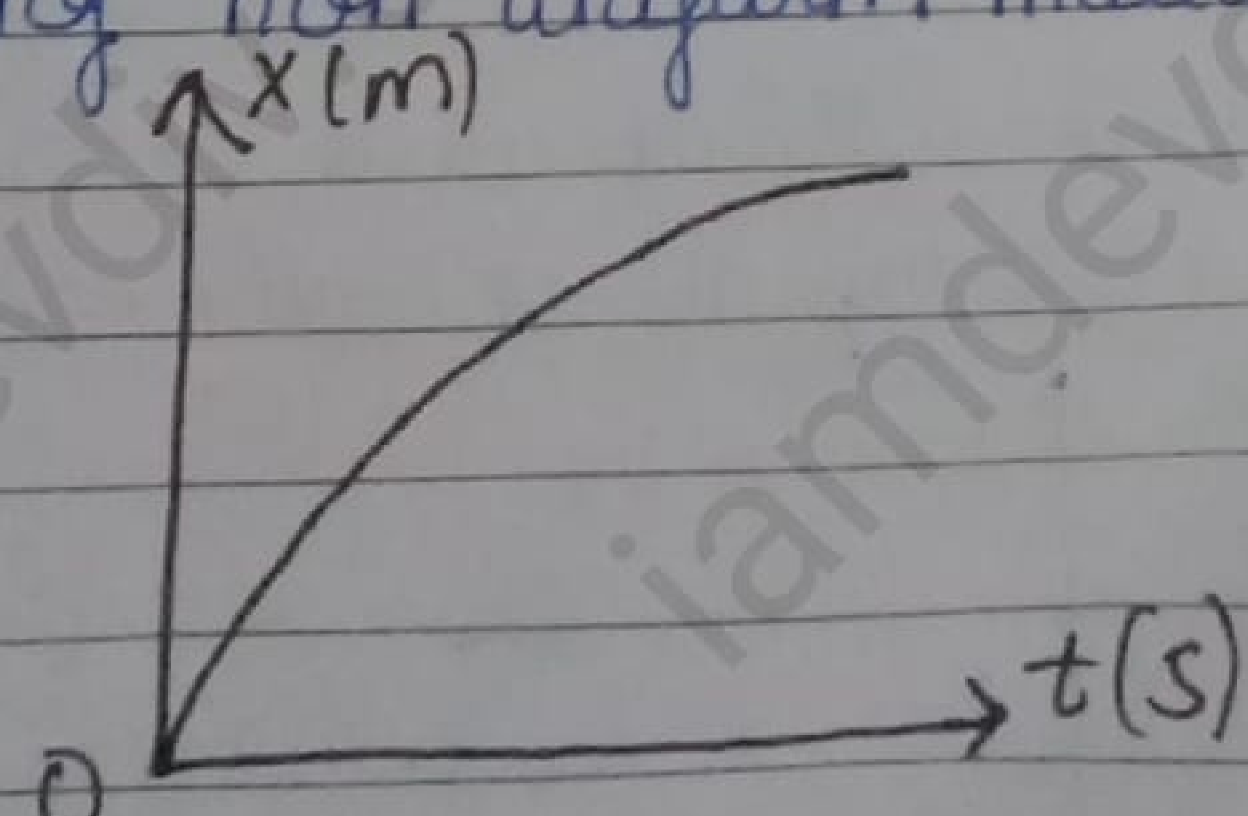
- The motion of an object can be graphically represented by position time graph. Position time graph of a stationary object is



- If an object is travelling equal distance in equal intervals of time, then it is said to be in uniform motion. The position - time graph of an object undergoing uniform motion is



- If an object is travelling unequal distance in equal intervals of time or vice versa, the object is said to be in non-uniform motion. The position - time graph for an object undergoing non uniform motion is



→ **Acceleration:-**

Acceleration of an object in motion is defined as the ratio of change in velocity and the corresponding time taken by the object

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

It is a vector quantity SI unit of acceleration is $m\,s^{-2}$. Its dimensional formula is $[M^0 L^1 T^{-2}]$

- **Uniform acceleration:-** An object is said to be moving with a uniform acceleration if its velocity changes by equal amounts in equal interval of time.

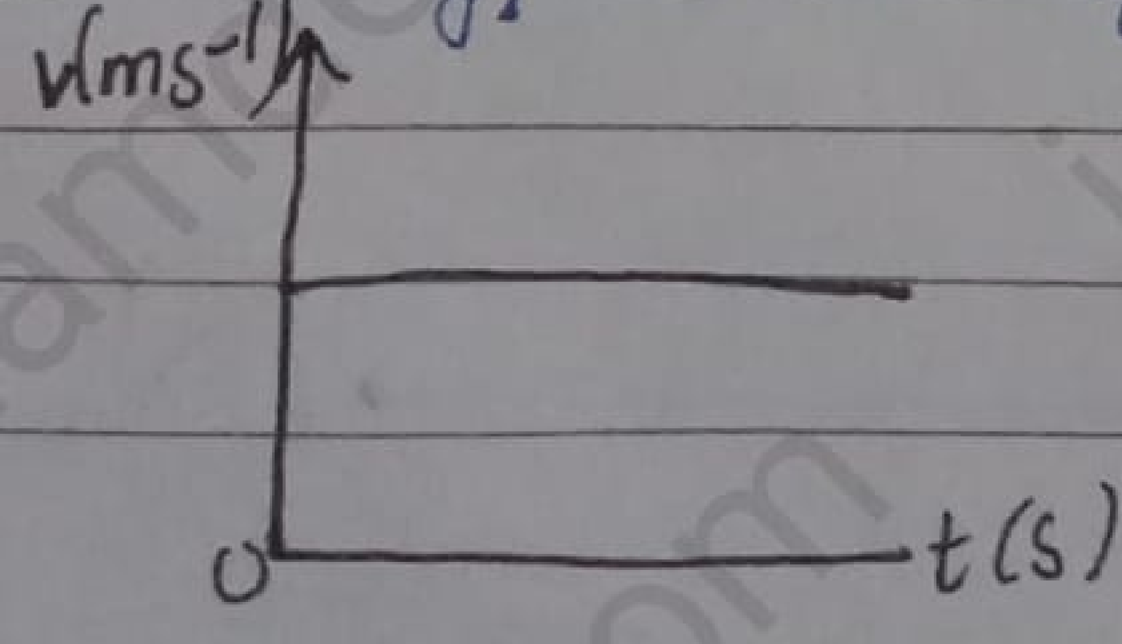
- **Average acceleration:-** It is the ratio of total change in velocity of the object during motion to the total time taken.

- **Instantaneous acceleration:-** The acceleration of the object at a given instant of time is called its instantaneous acceleration.

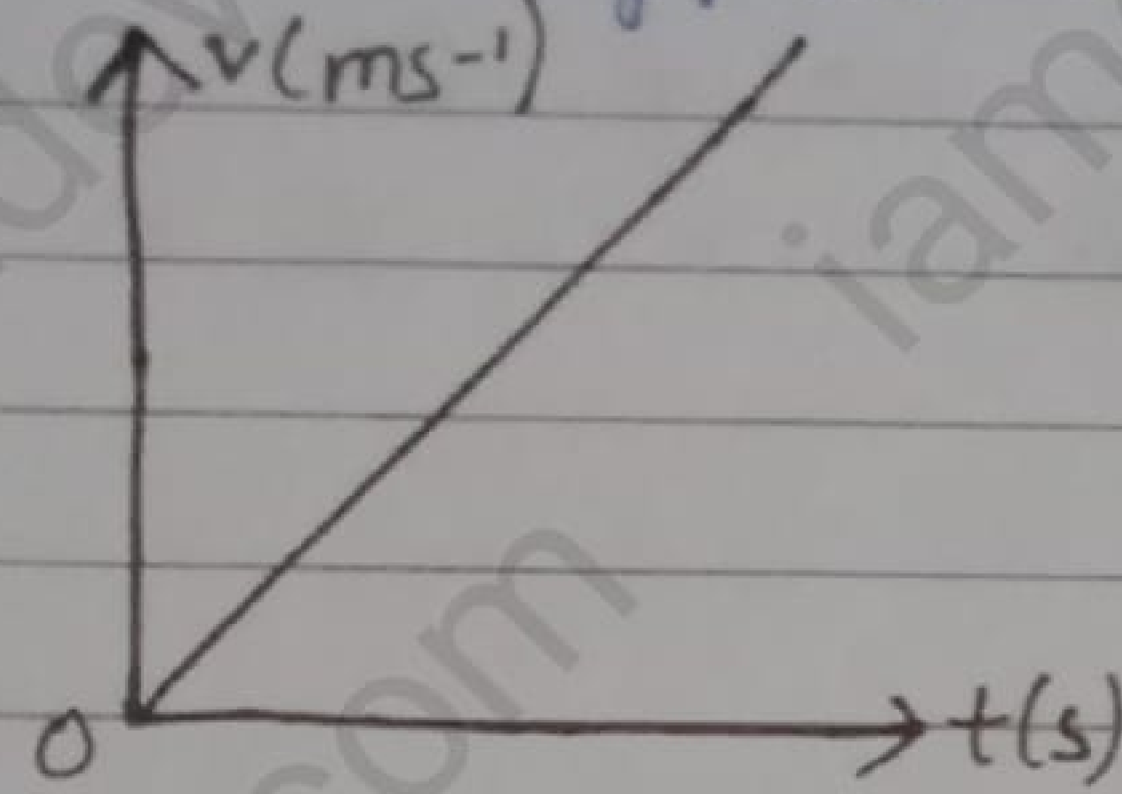
$$\text{Instantaneous acceleration} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

* **Velocity - time graph for an accelerated motion**

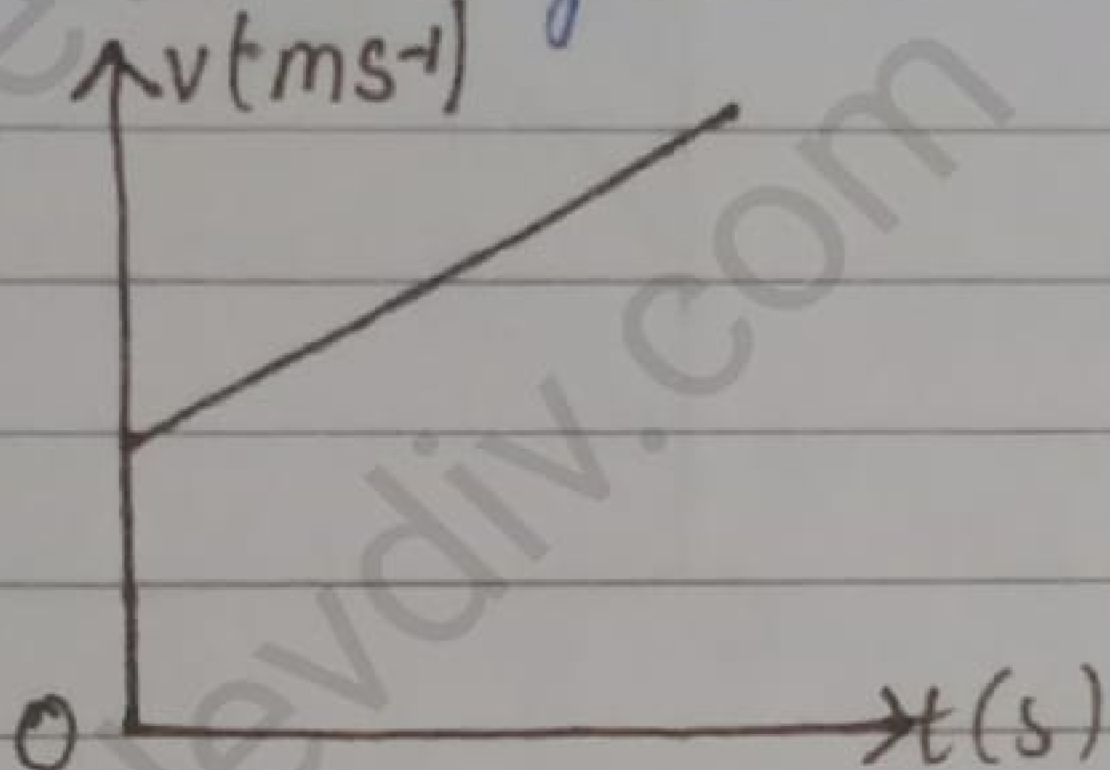
→ When an object is moving with zero acceleration, that is with constant velocity, the velocity - time graph is a straight line as



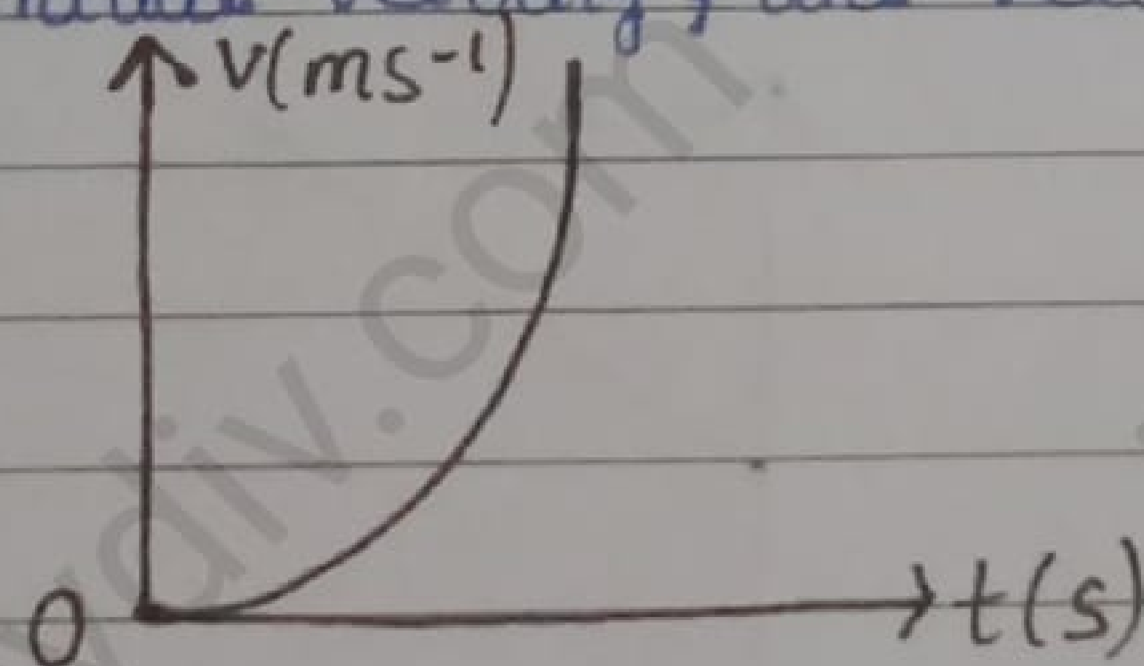
→ when an object is moving with constant positive acceleration, having zero initial velocity, its velocity graph is a straight line as



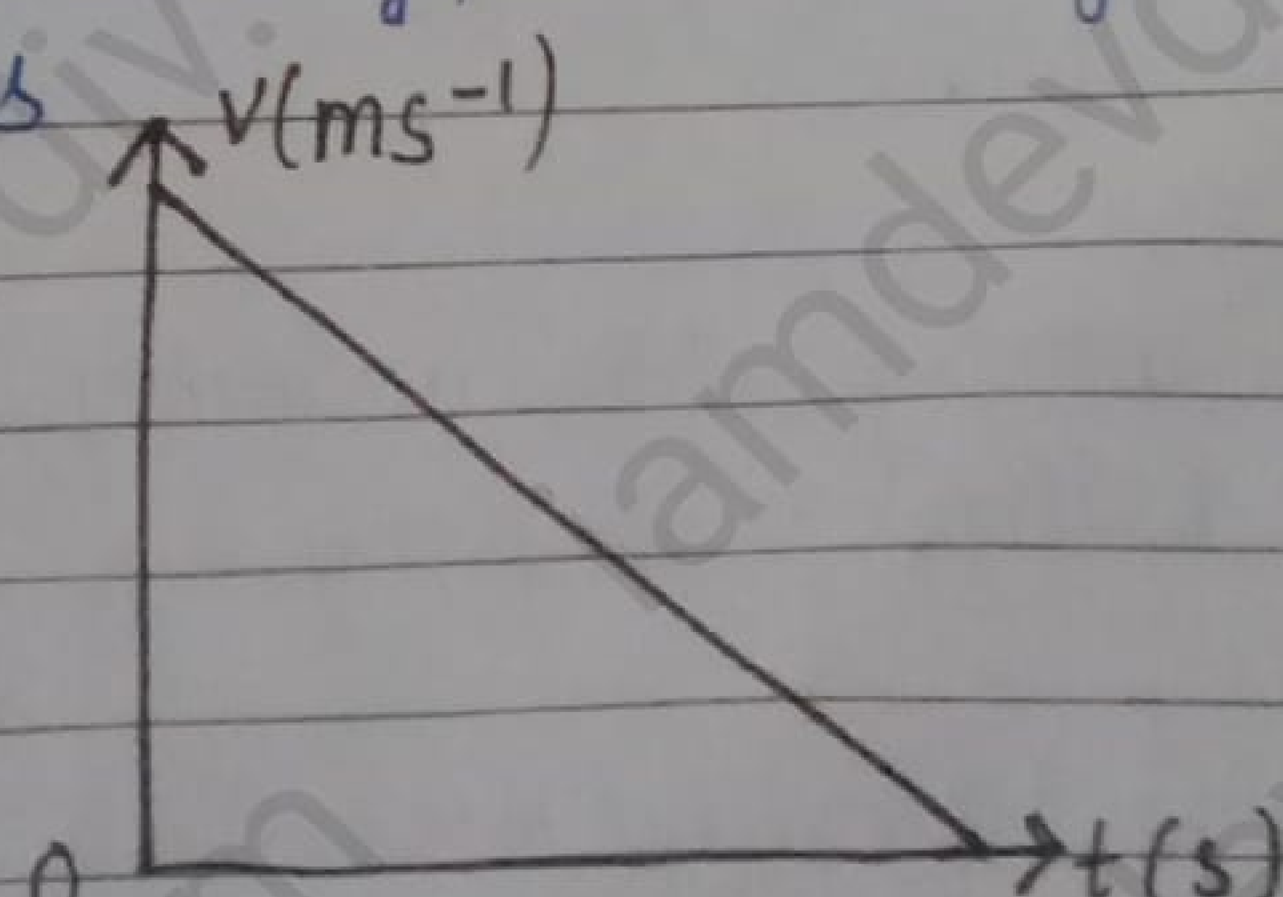
→ When an object is moving with positive constant acceleration with some initial velocity, the velocity-time graph is straight line as



→ When an ~~graph~~ object is moving with increasing acceleration, having zero initial velocity, the velocity - time graph is a curve as

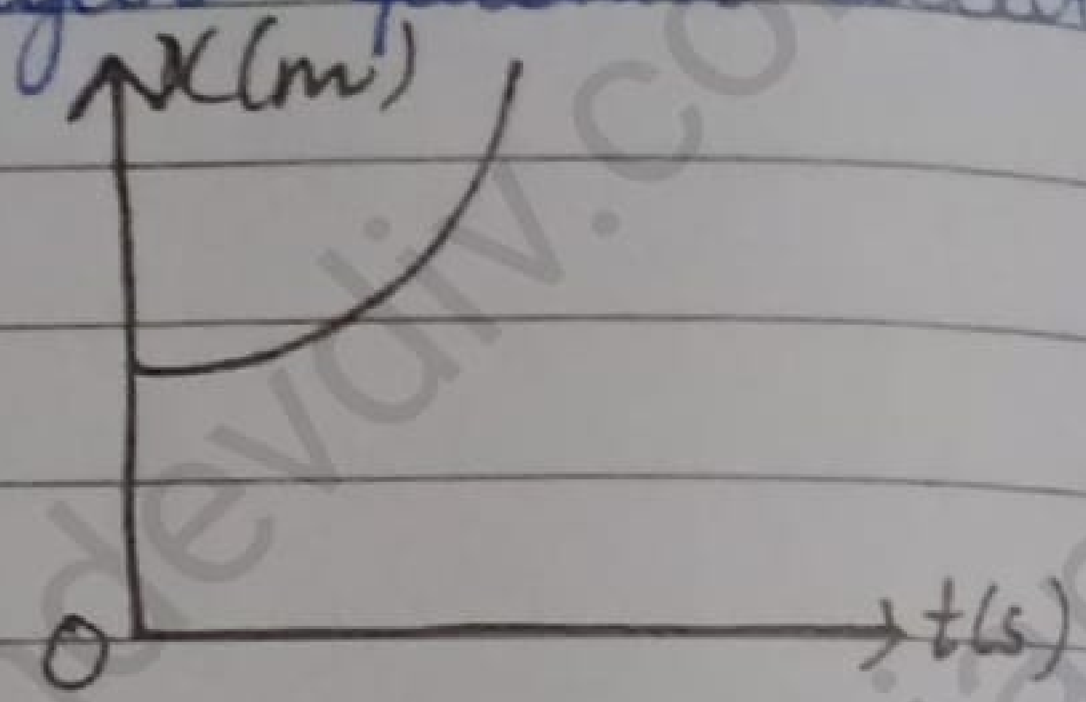


→ When an object is moving with constant negative acceleration having positive initial velocity, then velocity time graph is a straight line is as

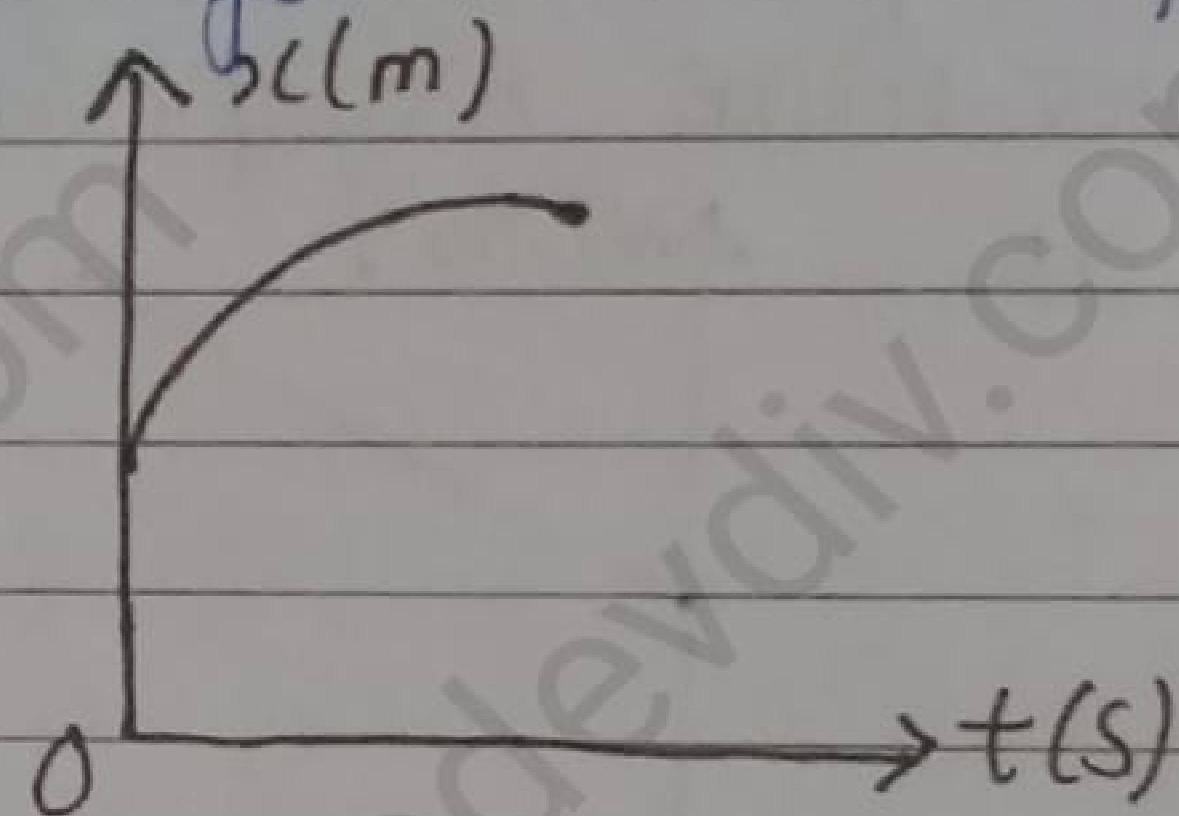


* Position - time graph for accelerated motion

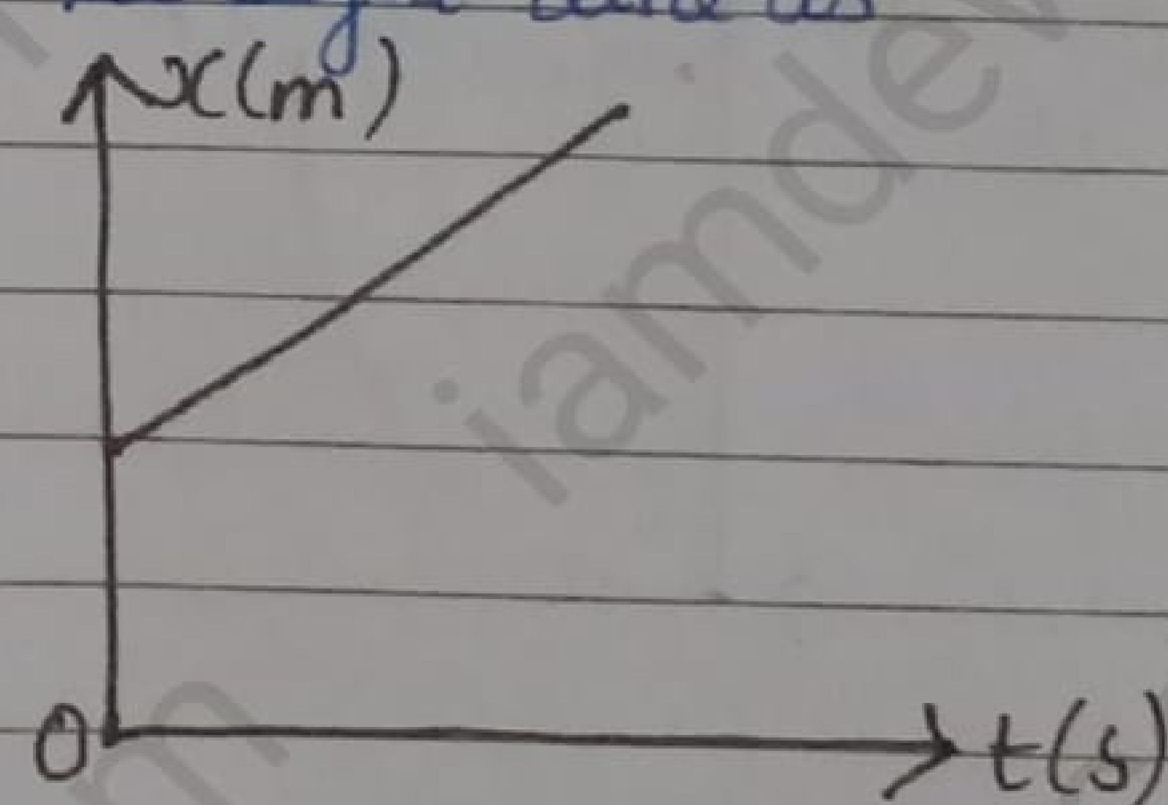
→ When and object is moving with uniform positive acceleration position - time graph is a curve as



→ When an object is moving with negative acceleration, then position time graph is a curve as



→ When an object is moving with negative acceleration, then position - time graph is a straight line as



* Elementary concept of differentiation and Integration for describing motion

→ If an object moving with velocity v along a straight line, the velocity at a time t is given by

$$V = \frac{dx}{dt}$$

where dx is the diplacement of the object

→ If an object is moving in a straight line with uniform acceleration, then $a = \frac{dv}{dt}$

where dv is the change in acceleration in time dt.

Also distance travelled by a particle

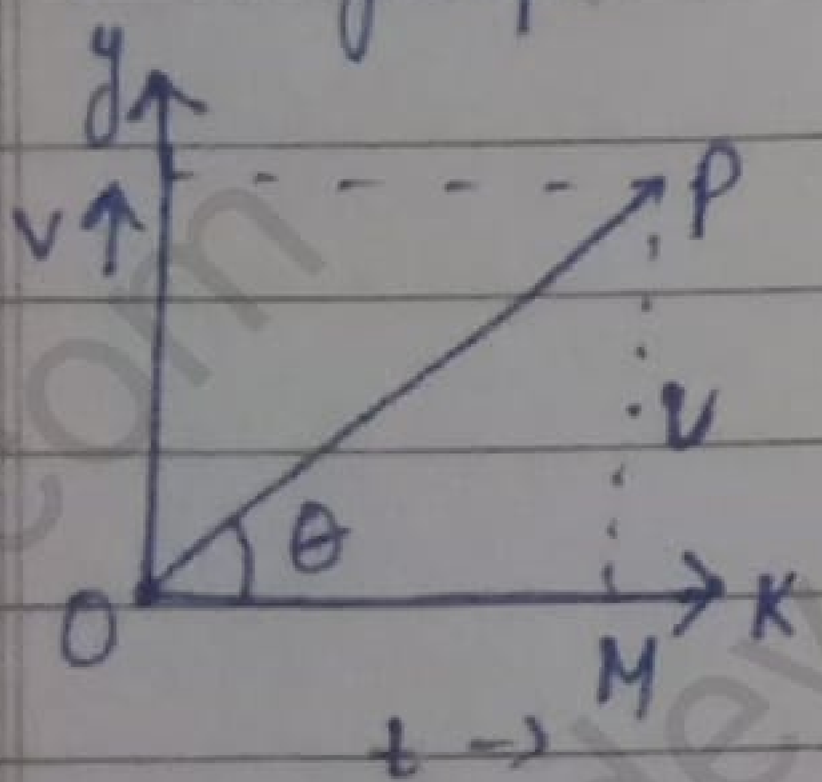
$$x = \int v dt$$

velocity of a particle,

$$V = \int a dt$$

$$\underbrace{V}_{\text{final velocity}} = \underbrace{u}_{\text{Initial velocity}} + \overbrace{a}^{\text{acceleration}}\underbrace{t}_{\text{time}}$$

v-t graph → Acceleration



Because we know that Acceleration $a = \frac{d}{dt} v$

ΔDPM

$\tan\theta = \frac{PM}{OM}$

$a = \frac{v}{t}$

$\frac{a}{t} = \frac{dv}{dt}$

$dv = 1dt$

$1dv = a\,dt$

$vdv = atdt$

$\int v^n dv = \frac{v^{n+1}}{n+1}$

Taking integration both side from eq (1)

$\int v^\circ dv = \int a t^\circ dt$ — (2)

velocity $\rightarrow$ u to v] limits
Time $\rightarrow$ 0 to t

$$\int_u^v v^\circ dv = a\int_0^t t^\circ dt$$

$$\left[\frac{v^{0+1}}{0+1}\right]_u^v = a\left[\frac{t^{0+1}}{0+1}\right]_0^t$$

$$\left[v\right]_u^v = a\left(t\right)_0^t$$

$$\left[v-u\right] = a\left[t-0\right]$$

$v - u = at \Rightarrow v = u + at$

Because we know that $v = \frac{ds}{dt}$

$ds = vdt$ — (1)

We know that
$V = u + at$ — (2)

from eq (2) & (1)
$ds = (u + at)\,dt$
$ds = udt + at\,dt$
$s^\circ ds = ut^\circ dt + at\,dt$
Taking integral both sides
$\int s^\circ ds = u\int t^\circ dt + a\int t\,dt$

$$\left[\frac{s^{0+1}}{0+1}\right]_0^s = u\left[\frac{d^{0+1}}{0+1}\right]_0^t + a\left[\frac{t^{1+1}}{1+1}\right]_0^t$$

$$\left[s\right]_0^s = u\left[t\right]_0^t + \frac{a}{2}\left[t^2\right]_0^t$$

$$[s-0] = u(t-0) + \frac{1}{2}a(t^2-0)$$

$$\boxed{s = u + \frac{1}{2}at^2}$$

3rd equation of motion

$$v^2 = u^2 + 2as$$

Because we know that $a = \frac{ds}{dt}$

$$\frac{a}{1} = \frac{dv}{dt} \qquad \frac{ds}{ds}$$

$$\frac{a}{1} = \frac{ds}{ds} \qquad \frac{dv}{dt} \qquad = \quad a\,ds = \frac{ds}{dt}\,dv$$

$$a\,ds = v\,dv$$

$$v\,dv = a^0 s\,ds$$

$$\int_u^v = v\,dv \quad a\int_0^s = s^0 ds$$

$$\left[v^2\right]_u^v = 2a\left[s\right]_0^s \Rightarrow v^2 - u^2 = 2as$$

$$v^2 = u^2 + 2as$$

4th Equation of motion

$$S_n = un + \frac{1}{2} a(2n-1)$$

$$D_n = un + \frac{1}{2} a(2n-1)$$

$$D_n = S_n - S_{n-1} \quad - (1)$$

$$S = ut + \frac{1}{2} at^2$$

if, $t = n$ sec. $\quad S_n = un + \frac{1}{2} an^2 \quad - (2)$

$t = (n-1) \quad S_{n-1} = u(n-1) + \frac{1}{2} a(n-1)^2 \quad (3)$

from eq (2) (3) & (1)

$$D_n = \left(un + \frac{1}{2} an^2\right) - \left[u(n-1) + \frac{1}{2} a(n-1)^2\right]$$

$$D_n = \left(un + \frac{1}{2} an^2\right) - u \cancel{\tfrac{1}{2}} (n-1) - \frac{1}{2} a(n-1)^2$$

$$= \cancel{un} + \frac{1}{2} an^2 - \cancel{un} + u - \frac{1}{2} a(n^2 + 1 - 2n)$$

$$= u + \frac{1}{2} \cancel{an^2} - \frac{1}{2} \cancel{an^2}(1 - 2n)$$

$$D_n = u + \frac{1}{2} a(2n-1)$$

Numericals

Q1- The displacement of a particle $=0$ when $t=0$ and displacement $=x$ when $t=t$. It start moving in the positive x direction with a velocity which varies $V=K\sqrt{x}$ where K is constant so that velocity vary with time

$$V = K\sqrt{x} \quad - (1)$$

$\downarrow$ velocity, $\downarrow$ constant, $\searrow$ distance

$t \to 0$ to t

$X \to 0$ to x

$V \to t$?

Because we know that $V = \frac{dx}{dt}$

from eq (1)

$$\frac{dx}{dt} = K\sqrt{x}$$

$$\frac{dx}{\sqrt{x}} = K dt$$

$$\therefore$$

$$x^{-\frac{1}{2}} dx = K t^0 dt$$

$$\int_0^x x^{-\frac{1}{2}} dx = K \int_0^t t^0 dt$$

$$\left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_0^x = K\left[\frac{t^{0+1}}{0+1}\right]_0^t$$

$$\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^x = k\left[t\right]_0^t$$

$$2x^{1/2} = kt$$

$$\sqrt{x} = \frac{kt}{2} \quad \text{— (2)}$$

from eq (2) & (1)

$$v = k\,\frac{kt}{2}$$

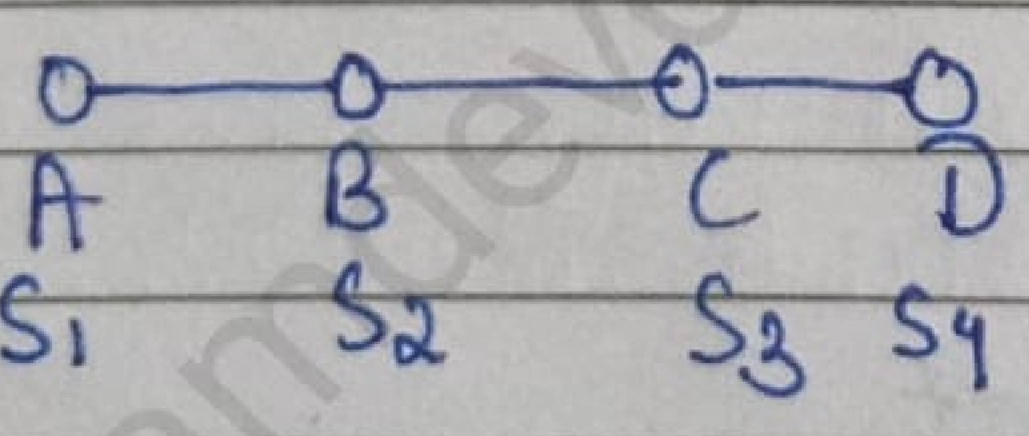
$$v = \frac{k^2 t}{2} \qquad v \propto t$$

~~It is the ratio of the average speed~~

→ Average speed

It is the ratio of total distance travelled by the object to the time taken.

$$V_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$



$AB = S_1 - t_1$
$BC = S_2 - t_2$
$CD = S_3 - t_3$

$$\boxed{V_{av} = \frac{S_1 + S_2 + S_3}{t_1 + t_2 + t_3}}$$